

Visualizing the Chances of Winning a Lottery

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Abstract

A hands-on sampling demonstration from a $C(13, 6)$ “lottery”, a column of beads with a three square foot base, and other examples will be used to demonstrate the slim chance one has of winning a lottery.

For those teaching counting principles at the high school or college level, a state lottery is a real-world example to which students can relate. The modern Louisiana lottery, Lotto, which ran from around January 1992 until December 1996, is an example of the combinations counting principle, $C(44,6)$, in which one is selecting six numbers from forty four in which repetition is not allowed and the order in which the numbers are selected is not important.

Although students recognize the number of possible selections is large (7,059,052) and the likelihood of winning the “jackpot” is small, some find it difficult to realize just how unlikely a jackpot is until comparisons with easier to visualize events are made.

To facilitate the student’s understanding of the magnitude of $C(44,6)$, the following activity has been developed and used at LSU-S:

LSU-S Lotto sampling activity. This activity consists of a fish bowl, a one-hole paddle, and $C(13,6) = 1,716$ approximately $\frac{1}{2}$ inch diameter beads, of which 1715 are the same color and one is of a different color. Students are told that each of the 1,716 beads represents a selection of six numbers from thirteen, and are then invited to “buy a lottery ticket” (take a sample of one bead). Although hopes for selecting the “winning” bead run high at first, they are quickly dashed as student after student fails to retrieve the “winning” bead.

Once students see first hand the difficulty of obtaining success with the LSU-S Lotto, the following examples can be used to illustrate the difficulty of winning an actual lottery, each of which relates the number of beads in the LSU-S Lotto to a familiar object, activity, or event:

Comparisons using surface area. The first of these comparisons involves surface area. The 1,716 LSU-S Lotto beads are approximately $\frac{1}{2}$ inch in diameter, so they occupy a surface of about 3 square feet, one layer deep, which is approximately 572 beads per square foot. Having a 1' x 3' piece of cardboard on hand helps the students grasp the three square foot concept. On the other hand, if the number of selections in the old Louisiana lottery were to be represented by $\frac{1}{2}$ " diameter beads, they would take up approximately 12,341 square feet, roughly equivalent to 123 10' x 10' bedrooms, a size typical for a small bedroom in a modest three bedroom house.

While visualizing 123 one hundred square foot rooms may be easy for some, those who are sports-minded may appreciate relating the probability of winning the old Louisiana lottery to the surface area of known athletic event sites. For example, the standard professional football field, which is 300 feet between the end zones and 160 feet between the sidelines, occupies 48,000 square feet. The 12,341 square feet of beads representing the old Louisiana lottery would take up about one fourth of the football field – roughly the distance between either end zone and the 25 yard line.

However, with regard to baseball, the same 12,341 square feet occupied by the old Louisiana lottery beads would not fit within the base lines in a major league baseball park. With a baseball diamond being a square 90 feet on a side, the area inside the base lines is approximately 8,100 square feet. One would need more than 1.5 baseball diamonds to contain the $\frac{1}{2}$ inch diameter beads representing the old Louisiana lottery.

Although basketball courts vary in size, the largest with dimensions of 94' x 50', occupies 4,700 square feet. More than 2.5 basketball courts of this size would be

necessary if one were to cover the playing surface with beads representing the old Louisiana lottery one layer deep.

Comparisons involving volume. Recalling that the LSU-S Lotto beads occupy a three square foot surface one layer deep, it would require 4,113 layers of these $\frac{1}{2}$ " diameter beads to represent the Louisiana lottery, resulting in a 171 foot high column with a three square foot base. Allowing 12 feet per floor, this column of beads would have a height of more than 14 stories.

A second volume example takes into consideration the space occupied by each of the beads. Having a diameter of $\frac{1}{2}$ " and a volume of $3.14159/48$ cubic inches, about 15.3 of these beads would be necessary to make up a volume of one cubic inch. (Of course, a cube one inch on a side would hold only eight $\frac{1}{2}$ inch diameter beads, with the remainder of the cube being empty.) With a gallon containing about 231 cubic inches, it would take the volume of 3,534 beads to make up a gallon. The volume occupied by the beads representing the old Louisiana lottery would total to about 1,997 ($7,059,0562/3,534$) gallons. Since, on average, a human being has about one gallon of blood, (adult males have about six quarts of blood, with women and children having less), the volume of beads representing the old Louisiana lottery would be approximately the same as the volume of blood in 1,997 human beings or in 1,331 ($1997/1.5$) adult males.

Comparison involving length. While the 1,716 LSU-S Lotto beads, if laid end to end, would stretch about 71.5 feet, by comparison, a string of beads representing the old Louisiana lottery would be considerably longer – 294,127 feet (55.7 miles).

Comparisons involving time. Selecting tickets from the LSU-S Lotto bearing all possible numbers within a week's time shouldn't prove too challenging. If this were an 8-hour-a-day job, one could select $(1,716) / (7 \times 8 \times 60) = .51$ sets of 6 distinct numbers per minute, or about one set every two minutes. On the other hand, to obtain all possible sets of 6 numbers in the Louisiana lottery within a week's time, it would require 2,101 selections of 6 per minute or 35 selections per second for seven 8-hour days. If one wished to make selections non-stop around the clock for a week, 11.67 different selections of six per second would be required.

All of the previous comparisons have been made using the old Louisiana lottery as a reference. With the current Louisiana lottery, a $C(40, 6) = 3,838,380$ scheme, the chances of winning are improved, meaning that comparisons, although dramatic, don't have quite the same impact. For example, the height of the column of beads with the three square foot base would be only 93 feet (7 + stories) tall, rather than the 171 foot tall column associated with the old Louisiana lottery.

In addition to the current Louisiana lottery, two additional lottery options are available to Louisiana residents – Louisiana Rolldown ($C(55, 5) = 3,478,761$) and Powerball ($C(49, 5) \times C(42, 1) = 80,089,128$). And, a recent Internet search yielded lottery activities available in 29 of the United States along with an adequate description of each state's game and the odds of winning. Many of these state lotteries are considerably more difficult to win than the current Louisiana lottery. Included in this group are the lotteries in Pennsylvania ($C(69, 6) = 39,959,158$), Massachusetts ($C(50, 5) \times C(36, 1) = 76,275,360$), and Texas ($C(54, 6) = 25,827,165$). The Excel spreadsheet program may

prove helpful for the student who wishes to examine comparisons with respect to time, volume, surface area, and length using other state lotteries as a reference point.